Overview of Mechanics

**Goal:** the goal of the science of mechanics is to discover concepts and principles enabling one to predict and explain motions of a system

**System:** a single particle or any specified set of particles

*Characteristics:* mass, charge, moment of inertia…

**Motion:**

*Descriptive Concepts:*

*basic:* position, velocity, acceleration

angular position, angular velocity, angular acceleration

*complex:* momentum, angular momentum, kinetic energy, center of mass

**Interactions:**

*Descriptive Concepts:* force, torque, work, potential energy

*Kinds of Interactions:*

*long-range interactions:* gravitational, electric, magnetic…

*contact interactions:* with spring, with string, with touching solid

**Mechanics Laws:** relations between a system’s motion and its interactions

*Basic Law for a Particle:* Newton’s law

*Laws for a System:* momentum law, angular-momentum law, energy law
# Motion of a Particle

## Descriptive Concepts

### Basic Concepts
- **Position vector**: \( \vec{r} \)
- **Velocity vector**:
  \[
  \vec{v} = \frac{d\vec{r}}{dt}
  \]
- **Acceleration vector**: \( \vec{a} = \frac{d\vec{v}}{dt} \)

## Rotational Motion
- **Angular position**: \( \phi \)
- **Angular velocity**:
  \[
  \omega = \frac{d\phi}{dt}
  \]
- **Angular acceleration**:
  \[
  \alpha = \frac{d\omega}{dt}
  \]

## Special Motions

### Straight-Line Motion
- \( a_x = \text{constant} \)
- \( v_x = v_{xo} + a_x t \)
- \( x = x_o + v_{xo} t + \frac{1}{2} a_x t^2 \)
- \( v_x^2 = v_{xo}^2 + 2 a_x (x - x_o) \)

### Circular Motion
- **Speed** \( v \), \( r \)
- **Acceleration components**
  - along \( \vec{v} \) (tangential):
    \[
    a_r = \frac{dv}{dt}
    \]
  - \( \perp \) to \( \vec{v} \) (centripetal):
    \[
    a_c = \frac{v^2}{r}
    \]
- **Period**:
  \[
  T = \frac{2\pi r}{v}
  \]
- **Frequency**:
  \[
  f = \frac{1}{T}
  \]

## Motions Relative to Different Frames
- for non-relativistic speeds \( (v << c) \)
  - notation convention: \( \vec{r}_{PA} = \) position of \( P \) with respect to \( A \)
  - **Positions**:
    \[
    \vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}
    \]
  - **Velocities**:
    \[
    \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}
    \]
  - **Accelerations**:
    \[
    \vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}
    \]
# Interactions

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Force</th>
<th>Potential Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direction</td>
<td>Magnitude</td>
</tr>
<tr>
<td><strong>Long-Range Interactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravitational</td>
<td>$\vec{F}$ vertically down (toward center of earth)</td>
<td>$F = mg$</td>
</tr>
<tr>
<td>due to earth, near surface</td>
<td></td>
<td>$(g = 9.8 , \text{m/s}^2)$</td>
</tr>
<tr>
<td>due to any particle</td>
<td>$\vec{F}$ attractive</td>
<td>$F = \frac{Gm_1m_2}{r^2}$</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Electric</strong></td>
<td>$\vec{F}$ repulsive for like charges attractive for opposite sign</td>
<td>$F = \frac{k_e</td>
</tr>
<tr>
<td>due to charged particle</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Contact Interactions</strong></td>
<td></td>
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<tr>
<td>Spring</td>
<td>$\vec{F}$ along spring toward the undeformed position</td>
<td>$F = -kx$</td>
</tr>
<tr>
<td>due to small deformation</td>
<td></td>
<td>$(F = 0$ if $x = 0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>String</td>
<td>$\vec{F}$ attractive</td>
<td>$F \neq 0$ if string is taut $F = 0$ if string is slack</td>
</tr>
<tr>
<td>due to tension force</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Touching Object</td>
<td>$\vec{F} = \vec{n} + \vec{j}$</td>
<td>$n \neq 0$ if touching $n = 0$ if not touching</td>
</tr>
<tr>
<td>normal force</td>
<td>$\vec{n}$ perpendicular to surface, opposes compression</td>
<td></td>
</tr>
<tr>
<td>friction force</td>
<td>$\vec{j}$ parallel to surface, opposes relative sliding</td>
<td></td>
</tr>
<tr>
<td>kinetic (relative motion)</td>
<td>$\vec{j}$ opposite relative velocity</td>
<td>$f = \mu_s n$</td>
</tr>
<tr>
<td>static (no relative motion)</td>
<td>$\vec{j}$ so that rel accel = 0</td>
<td>$f \leq \mu_s n$</td>
</tr>
</tbody>
</table>
### Momentum Law (Conservation of Linear Momentum)

**Validity**
- for inertial frame

**Utility**
- relates motion & external interactions
- relates velocities at any two instants if total external force is zero

\[
\frac{d\vec{P}}{dt} = \vec{F}_{\text{ext}}
\]

\[m\ddot{\vec{a}} = \vec{F}_i\] for particle

\[M\ddot{\vec{A}}_C = \vec{F}_{\text{ext}}\] for center of mass

### Angular Momentum Law (Conservation of Angular Momentum)

**Validity**
- for inertial frame
- relative to CM always

**Utility**
- relates rotation & external interactions
- relates angular velocities at any two instants if total external torque is zero

\[
\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{ext}}
\]

\[I\ddot{\vec{\alpha}} = \vec{\tau}_{\text{ext}}\] for particle if \(I\) is constant

### Energy Law (Conservation of Energy)

**Validity**
- for inertial frame

**Utility**
- relates speeds & positions at any two instants (without mention of time)

\[
\Delta E = W_{\text{oth}}
\]

\[TME = KE + U\]